The Doubling Strategy in Backgammon

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It is basically assumed that you are familiar with The Rules of Backgammon and the most common Backgammon Glossary.

1. When you are presented to a Double

When you are presented to a double you got two possibilities leaving out *beavers*. You can accept or reject (take or drop).

- If you reject you loose the value (say 1) of the cube before it was given.
- If you accept the game continues. If you do not redouble and there is no *gammon or backgammon* you will win or loose the new value (say 2) of the cube.

This gives the following probabilistic considerations. The easiest way when considering to accept a cube is to evaluate from the situation where you loose the former value of the cube (say 1) i.e. the drop situation.

- If you accept and loose you really only loose half the value (say 1) of the cube.
- If you accept and win you really win one and a half the value (say 3) of the cube.

If risk considerations are ruled out by assuming risk neutrality this means that you want to accept the cube when you have 1/(1+3)=25% (your *take-point*). The reason is that by accepting the cube you risk half the value (say 1) of the cube to have the chance of gaining one and a half the value (say 3) of the cube.

Another way to illustrate this very important point is to imagine that you play 4 games. In each game exactly the same position arise in which you have 25% winning chances (no cube evaluation) and the cube is still centered at 1. All of the 4 games your opponent doubles. If you drop in all the games you lose 4 points. Now if you take all 4 cubes and do not redouble you will on average win 2 points in one of the games and lose 2 points in three of the games. This means that in all the 4 games will lose 6-2=4 points. This means that if you do not redouble you have a border-line decision when you have 25% winning chances.

In the considerations above I used a number of assumptions.

- First I did not consider *beavers*.
- Second I did not consider redoubles. This meant that you never exploited *the value of holding the cube*.
- Third I did not consider *gammons and backgammons*.
- Finally I assumed risk neutrality. I will not comment on this assumption since small distortions in risk attitude can be very bad for your game. If you have problems with risk attitude I suggest you play for less money.

1.1 Beavers

Basically you want to Beaver when you are ahead in the game because the decision does not influence the optimal plays. You can even beaver when you are a little behind in the game as I will comment on in *the value of holding the cube*.

1.2 The value of Holding the cube

When you get the cube you will be the only one that have the option of doubling. This has non-negative value since you get more possibilities in your game and because you can always omit to double. The reason the value is positive in non-trivial positions (i.e. not last roll positions or positions with no winning chances) can be illustrated by the following little chain of thoughts.

Let us say you have accepted a cube and have 24% winning chances. Then in a little more than those 24% of the games you will at some point reach a position where you have more than 80% winning chances. If you then give the cube your opponent's best play is to reject. This "steals" away some of his winning chances (that was considered in the original 100%-24%=76% winning chances) since he in these positions will not be able to win the less than 20% of the games he would win without the cube.

When considering *beavers* this effect has a larger impact since there is much longer from 24% to 80% than from e.g. 49% to 80%. This means that you much more often will get the opportunity of "stealing" some of your opponent's winning chances.

Of course this reasoning works both ways. I.e. it has negative value for you when your opponent holds the cube.

It is not the best strategy always to wait until you have more than 80% winning chances to give the redouble. More about this in *when to double*.

1.3 Gammons and Backgammons

Gammons have a very big impact on cube decisions. This can be illustrated by the following calculations.

Consider a position where you have 33% winning chances and no Gammon or Backgammon chances but your opponent is winning a Gammon in 33% of the games and no Backgammons. Leaving out redoubles (see *the value of holding the cube*) by accepting a cube on 2 you will lose 33% * 4 + 34% * 2 - 33% * 2 > 1. This means that you are better of rejecting a cube even though you have 33% winning chances.

The formula I used is very simple and the general thing to use when considering a double theoretically (still leaving out redoubles). Before formulating it generally I need the following definitions.

- x_w = Your normal winning chances (no gammon or backgammon)
- x_g = Your gammon chances
- x_b = Your backgammon chances
- y_w = Opponent's normal winning chances
- y_g = Opponent's gammon chances
- y_b = Opponent's backgammon chances

Formula for your equity:

$$x_w * 1 + x_g * 2 + x_b * 3 - y_w * 1 - y_g * 2 - y_b * 3$$

The number you get from this formula is the expected number of points you win (your equity). If 2 times this number is larger than -1 (your equity > -0.5) you should accept if you are doubled. As previously discuses the matters are not that simple if the reverse is true (see *the value of holding the cube*).

Although the formula is simple it is very hard to use it over the board. Therefore most players choose to develop an intuition and use heuristics to decide when to accept or reject a double rather than trying to use the exact formula.

1.4 Take-point

The analysis will get much more complicated, so it is a good idea to have some simplifying vocabulary. The notion of a take-point is exactly such a thing.

A take-point is the winning chances you will have to have to be able to accept a cube. As mentioned earlier this is basically 25% in money game but as discussed earlier both *the value of holding the cube* and *gammons and backgammons* may influence the breaking point of when to accept or reject a double.

Considering *the value of holding the cube* you have to make an adjustment to less than 25%. This will depend on how volatile the position is and this adjustment is very complex. Only heuristics can aid your estimate here. Besides this the adjustment is so small that the uncertainty in your evaluation of your winning chances will make spending much effort adjusting your take-point this way ridiculous.

Considering *gammons and backgammons* in the take-point is also very hard since the take or drop of a cube depend on six inputs namely x_w, x_g, x_b, y_w, y_g, and y_b (They sum to 100%). These inputs are very hard to measure in one number. Backgammon is a very complex game and therefore it may be a good idea to simplify matters. The first thing most people do is to ignore backgammons. This is actually half the step to only use the take-point. Therefore the take-point is often but not always used even in positions where *gammons and backgammons* matter. Players then just roughly adjust the take-point for gammons when evaluating the position or simply ignores *gammons and backgammons* (this is very dangerous since they have a big impact on the correct cube handling).

Computers use equity in stead of winning chances which the take-point is based on. An equity is the expected number of points won in the position. It is sometimes adjusted for *the value of holding the cube*. This approach is very hard to use for a human because chances and risks are much more intuitive and easy to interprete.

The notion of a take-point becomes even more important and complex in *match play* when considering *the match take-point*.

2. When to Double?

Basically you would like to play for more points when you are ahead in the game (see *beavers*). Although this may seem to make doubling decisions simple it does not. The main point is that you give up your option of doubling later and for reasons stated earlier (see *the value of holding the cube*) this has a non-positive value on your winning chances (unless your opponent rejects the cube). This means that you given your opponent are going to accept would like to wait as long as possible with giving him the cube. The "problem" is that your opponent need not always accept the cube. This makes *market losers* a very important issue.

2.1 Market Losers

When you consider if you should double you should consider what will happen during the next exchange of rolls. If your opponent have no chance of reaching a position where he has a drop you should never double because there is no way you can lose your market. This means that by doubling you will just give up some winning chances without gaining anything in return because you will have the option of giving the cube on your next turn.

The thing you gain by doubling is the difference between your opponent's *take-point* and his winning chances after a market loosing exchange of rolls.

This means that the doubling decision depends on the trade-off between the winning chances you give up by handing over the cube and your opponent's expectet loss due to rolling a market loosing exchange after his take.

Therefore the volatility of the position is important to your cube action. A position with a small volatility is almost never a double/take since the market losers are very small (if there are any).

2.2 Sufficient conditions

Over the board it can be very hard to use the above approach. This means that only heuristics can aid you when you are considering a double. The thing I will present here is some sufficient conditions for not wanting to play for more points. These are exact and will not be fulfilled when you have a double according to the above mentioned trade-off.

$$x_w * 2 + x_g * 4 + x_b * 6 - y_w * 2 - y_g * 4 - y_b * 6 < x_w * 1 + x_g * 2 + x_b * 3 - y_w * 1 - y_g * 2 - y_b * 3$$

This simply reduces to considering whether you are behind in the game. If the above formula is adjusted for changes in the winning chances by handing over the cube the left side of the inequality sign will decrease. Non gammonish last roll positions will be a double if the inequality is not fulfilled!

Again in *match play* this inequality becomes much more complex and important because a match equity table is used when you consider *when to double in matches*.

2.3 The Doubling Window

With correct play the above considerations will often lead to positions where you have a correct double and your opponent will have a take but over the board it can be very hard to use the above approach. The doubling-point is an easier heuristic that rely on a simplification of the *sufficient conditions*. Analog to the *take-point* above rough adjustments for gammons and backgammons is made to the overall winning chance needed to want to play for more points (approximately 50%). Again it is important to stress that this is a necessary and not sufficient condition.

The space between the *take-point* and the doubling-point is called the doubling window. Roughly speaking you need to be inside the doubling window to have a correctly given accepted cube. Again the notion of a the doubling window becomes more important in *match play* where you consider *the match doubling window*.

2.4 Too good

If you wait to long before doubling or an extreme *market loser* is rolled a position can be reached where you should not double because your position is too good. This can easily be seen by the following example.

You have reached a position where you have a 50% gammon chance and no backgammon chance. Your opponent is struggling with a 25% (no cube) winning chance and no gammon or backgammon chance. If you double here you will surely win the value (say 1) of the cube but by never doubling you will also expectedly win the value of the cube. You will have the option of doubling later and thereby "stealing" some of your opponent's winning chances witch makes it better to wait with the double. Actually the option of doubling later will probably drive your opponent's winning chances down to less than 5% and therefore making it a big mistake to double.

The 50% gammon versus 25% loosing chance was border line. This can be genneralized to whenever you have twice as much gammon chance as your opponent's winning chance. In this case it is a sufficient but not nessesery condition. This fact also give us a nice little heuristic that are useful when choosing between a safe move and a risky but gammonish move. If you give up some winning chance to gain a larger chance of winning gammon this will only be correct if you gain twice as much gammon chance as the winning chance you give up. To realize this imagine that you win all of the games normally. If you win gammon you will win one point more. If you loose you loose 2 points compared to your normal win situation.

You should be very careful when playing a too good position because big mistakes are usually made here (many mistakes are usually made playing on in a not too good situation). It should be said that you can even play on when your expected winnings are less than the value of the cube. The thing to consider are market gainers (the opposite of *market losers*). If you have a gammon chance and no market gainers you will not have a double no matter how small the gammon chance is.

2.5 Centered cube

The initial double is special in the respect that you do not give your opponent the option of doubling since he has already got it (both players have *value of the cube*). You only take away your own option. This gives a slightly more aggressive cube handling since the loss in winning chances from handing over the cube is smaller when it is centered (see the above mentioned *trade-off*).

In money game most players use the Jacoby rule. This rule will distort things because you do not use *gammons and backgammons* in your non-doubling calculations. Of course this may further advance the initial double because the part that considers doubling normally has the larger *gammon and backgammon* chances.

3. The cube in Matches

As mentioned earlier cube handling in matches are much more complicated. The reason is that you are now trying to maximize your chance of winning the match rather than just maximizing the expected number of points that you win. Maximizing your match winning chances (MWC) make some points more important than others.

The easy example is that when you are one point from winning the match you do not care if you win one or two points. The second point have no value at all. It is easy to see that this phenomenon also have an influence at other match scores. *Gammons and backgammons* makes the handling even more complex. Although these distortions change the game most of the above guidelines are still relevant to match play.

In matches the important thing to remember is how many points you and your opponent have left before a winner is found. It does not matter whether you are ahead 5 to 4 in a 9 point match or are ahead 1 to 0 in a 5 point match. In both situations you are ahead 4 away to 5 away.

A special rule that matches are played with is the Crawford rule. It is applied because the match score leader will face a smaller value of the lead than is reasonable.

3.1 The Match value of Holding the cube and Mandatory doubles

As in money game it often still have a *value to hold the cube*. Sometimes though the cube dies. If you are ahead 2 away to 4 away and takes an initial cube you will never redouble since you can never improve your MWC this way. In this situation it does not have any value that you hold the cube. The only indirect value is that your opponent does not have any value by the option of doubling.

Sometimes the cube is so much alive that you have a mandatory double. This means that you should double on the first legal opportunity. If your opponent will win the match if he wins this game you might as well give him the cube now. In matches it does not matter whether your opponent overshoots the target. In a 5 point match a victory on 5-0 is as good as one on 9-0.

On last roll positions your opponent will not get a chance to give his mandatory redouble. This makes the cube handling much more aggressive.

3.2 The Match Take-point

Normally the *take-point* is calculated ignoring *gammons and backgammons* and the redouble option. This is also true for matches. The way that the problem is approached is to evaluate your MWC at three relevant senarius:

1. Your MWC if you drop.
2. Your MWC if you take and lose this game.
3. Your MWC if you take and win this game.

To calculate your match take-point you simply use the formula (using the above given three relevant senarius):

$$\text{Take-point} = \frac{\text{risk}}{\text{risk+gain}} = \frac{\#1 - \#2}{\#1 - \#2 + \#3 - \#1}$$

This formula can be generalized to include *gammons and backgammons* but still ignoring the redouble option. We will need the following data:

1. Your MWC if you drop.
2. Your MWC if you take and lose.
3. Your MWC if you take and lose gammon.
4. Your MWC if you take and lose backgammon.
5. Your MWC if you take and win.
6. Your MWC if you take and win gammon.
7. Your MWC if you take and win backgammon.

As in *the money game section* I define:

- x_w = Your normal winning chances in this game (no Gammon or Backgammon)
- x_g = Your Gammon chances
- x_b = Your Backgammon chances
- y_w = Opponent's normal winning chances
- y_g = Opponent's Gammon chances
- y_b = Opponent's Backgammon chances

To calculate when you have a take (the enhanced take-point) you use the following formula (using the above given seven relevant senarius):

$$x_w * (\#5 - \#1) + x_g * (\#6 - \#1) + x_b * (\#7 - \#1) > y_w * (\#1 - \#2) + y_g * (\#1 - \#3) + y_b * (\#1 - \#4)$$

It is easy to include *mandatory redoubles* in the above formulas.

As for the *money game take-point* if this formula is fulfilled you should accept if you are doubled. As previously discuses the matters are not that simple if the reverse is true (see *the match value of holding the cube and mandatory redoubles*). If the cube dies (maybe via a *mandatory redouble*) the formula including *gammons and backgammons* will be exact! I.e. you have a take if and only if it is fulfilled. If you like you can reduce the formula to the simple no *gammon or backgammon* version by assuming $x_g=x_b=y_g=y_b=$"0" and $y_w=(1-x_w)$.

The take-point viewed from the other side of the board is called the cash-point. I.e. cash-point = 100% - take-point.

Normally the MWC for various scores is found in a tabula e.g. Kit Woolsey's match equity table. This table is quite old and most computer programs use newer tables. Some of these can be found via the source of GNU Backgammon.

Some people like to use approximating formulas rather than a table. I do not recommend this since the approach is less precise and a table is more intuitive.

3.3 When to Double in Matches?

This question is one of the most complicated in backgammon. As in money game *market losers* play a key role here. This has to be measured up against the roll sequences where you wish you had not doubled (and did not double) including MWC and *the match value of holding the cube*.

If you lose your market after you doubled you have gained the difference between your expected MWC (including the uncertain outcome of this game) and your MWC given your opponent dropped the cube. If you doubled and experience a sequence of rolls where you wish you had not doubled you have lost some MWC.

This means that the doubling decision depends on the trade-off between the MWC you give up by sequences of rolls where you wish you had not doubled and your opponent's loss in MWC due to rolling a market loosing exchange after his take.

Sufficient conditions for not wanting to play for the double amount of points (the match double-point) can be found ignoring *gammons and backgammons* and the later redouble option. The way that the problem should be approached is to evaluate your MWC at four relevant senarius:

1. Your MWC if you double and win this game.
2. Your MWC if you double and lose this game.
3. Your MWC if you do not double and win this game (equals your MWC if your opponent drops).
4. Your MWC if you do not double and lose this game.

Note that you have already used three of these when you calculated the *match take-point*. To calculate your match double-point you simply use the formula (using the above given four relevant senarius):

$$\text{Double-point} = \frac{\text{risk}}{\text{risk+gain}} = \frac{\#2 - \#4}{\#2 - \#4 + \#1 - \#3}$$

As you can see your opponent's match doubling point is 100% minus your doubling point. The formula can be generalized to include *gammons and backgammons* but still ignoring the redouble option. We will need the following data:

1. Your MWC if you double and win this game.
2. Your MWC if you double and win gammon this game.
3. Your MWC if you double and win backgammon this game.
4. Your MWC if you double and lose this game.
5. Your MWC if you double and lose gammon this game.
6. Your MWC if you double and lose backgammon this game.
7. Your MWC if you do not double and win this game (equals your MWC if your opponent drops).
8. Your MWC if you do not double and win gammon this game.
9. Your MWC if you do not double and win backgammon this game.
10. Your MWC if you do not double and lose this game.
11. Your MWC if you do not double and lose gammon this game.
12. Your MWC if you do not double and lose backgammon this game.

To calculate when you do not want to play for the double amount of points (the enhanced double-point) use the following formula (using the above given twelve relevant senarius):

$$x_w * (\#1 - \#7) + x_g * (\#2 - \#8) + x_b * (\#3 - \#9) < y_w * (\#10 - \#4) + y_g * (\#11 - \#5) + y_b * (\#12 - \#6)$$

It is easy to include *mandatory redoubles* in the above formulas. If you like you can reduce the formula to the simple no *gammon or backgammon* version by assuming $x_g=x_b=y_g=y_b=$"0" and $y_w=(1-x_w)$. It is important to stress that the formula is only sufficient conditions for not wanting to play for more points. This means that if you have a last roll position and the formula is not fulfilled you have a correct double (if you are not *too good*).

This limit can work as an important guide line when you evaluate how much MWC you give up by doubling and experience a sequence of rolls where you wish that you had not doubled. Another limit that some people like to use is there own *match take-point* or the opponent's match (re) double-point given that the opponent takes a cube now. The relevant question they ask them self is how far does the opponent have to turn the game around before he can consider a redouble. This of course tell them something about what they give up by doubling now.

The initial double is also special at some match scores since your *match take-point* at the current cube level 1 is also relevant in these situations. As in money game this theoretically gives a little more aggressive *initial cube handling* than if you hypothetically owned the cube at level 1.

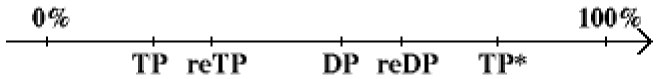
3.4 The Match Doubling Window

We have some simplifying vocabulary that can ease communication (important to learning) about *cube handling in matches*. The *doubling window* is exactly such a thing. It is defined as the interval between the *match take-point* and the *match double-point*. It is easy to realize that this can be generalized to include *gammons and backgammons* (no longer as an interval) and *mandatory redoubles*.

The doubling window is just a heuristic. Many other heuristics may aid you in your search for the correct cube handling. To list some of them I need some definitions:

- TP = your *take-point* before you have doubled.
- reTP = your *take-point* if your opponent redoubles after you have doubled.
- DP = your *double-point* now (equals 100% minus your opponent's *double-point*).
- reDP = your *double-point* after the game has been doubled (equals 100% minus your opponent's *double-point*).
- CP = your opponent's *take-point* now seen from your side of the board (your cash-point which equals 100% minus your opponent's *take-point*).

If you are the leader in the match the relevant (take-/double-) points will often look something like this (where TP*=CP):



The doubling windows (intervals) are:

- [DP;CP] = Your doubling window! This window does not contain any information about redoubles but only the distance to the point where you do not want to play for the double amount of points.
- [TP;DP] = 100% minus your opponent's doubling window now.
- [reTP;reDP] = 100% minus your opponent's doubling window after he has taken.

I suggest that you keep it simple by focusing primarily on your own doubling window but other relevant windows (intervals) are:

- [reDP;CP] = Your redoubling window. For mandatory redoubles this window is absurd. The reDP will then normally equal 100%. The window can sometimes be of some use since it tells something about how soon you can expect a redouble.
- [TP;CP] = The market window. Well it tells you where you can expect takes (including your own).
- [reTP;CP] = Your re-market window. As for the redoubling window this window can be of some use since it tell something about how soon you can expect a redouble.
- [TP;reTP] = Your initial take-point window. This window tells you something about how much more aggressive you should handle initial cubes.

3.5 Too good in Matches

The evaluation of when you are *too good* in matches is not surprisingly more complex than in money game. To give an understanding of the issue the first thing I will do is ignore backgammons and the opponent's gammon chances ($x_b=y_b=y_g=$"0"). The following MWC is needed:

1. Your MWC if you double and win this game.
2. Your MWC if you double and win gammon this game.
3. Your MWC if you double and lose this game.
4. Your MWC if you do not double and win this game (equals your MWC if your opponent drops).
5. Your MWC if you do not double and win gammon this game.
6. Your MWC if you do not double and lose this game.

With this the too good-point and the enhanced *take-point* (both is not points like the others but is derived in a similar manner; hence the names) can be reduced to the following formulas (using the above given six relevant senarius).

The too good-point:

$$x_w * (\#4 - \#4) + x_g * (\#5 - \#4) > y_w * (\#4 - \#6)$$

The opponent's enhanced *take-point*:

$$y_w * (\#4 - \#3) > x_w * (\#1 - \#4) + x_g * (\#2 - \#4)$$

It is easy to see that if the first formula (the too good-point) is fulfilled you do want to play on for gammon. Even when this formula is not fulfilled it is sometimes correct to play on for gammon. The important thing to observe is market gainers (the opposite of *market loosers*). This has to be weighted up against the gain in MWC by moving past the too good-point.

This means that the too good decision depends on the trade-off between the MWC you gain by sequences of rolls where you move past the too good-point and the loss in MWC due to rolling a market gaining exchange now.

The too good-point can also be used in another way. If you are too good and have a checker play decision where you can increase your gammon chance and your opponent's winning chance by playing bold what will be the correct play? The answer is that if you increase your gammon chance times (#5 - #4)/(#4 - #6) more than your opponent's gain in winning chances the bold play is correct. (#5 - #4)/(#4 - #6) is called the gammon price or gammon value depending on which side of the board you look at it from.

The gammon price is calculated as follows.

$$\text{Gammon price (GP)} = \frac{\text{gammon risk}}{\text{winning gain}} = \frac{\#5 - \#4}{\#4 - \#6}$$

This meens that the gammon price is the cost of getting gammoned relative to the gain from winning. The gammon value is the gain from winning gammon relative to the risk of loosing. I.e. your gammon price is your opponent's gammon value and the other way around.

The gammon price can be used to ajust your *take-point*. If you imagine a position where you do not have a gammon chance ($x_g=x_b=0$) your self and your opponent have a gammon chance of y_g ($y_b=0$). What will your winning chance (x_w) have to be before you have a take? The answer is:

$$\text{Gammon adjusted take-point } (GTP(y_g)) = TP + GP * y_g$$

where TP is the normal *take-point* and GP is your gammon price after a double/take. You can check this by assuming $x_g=x_b=y_b=0$ and $y_w=1-x_w-y_g$, and isolate x_w in the enhanced *take-point*.

3.6 Examples

Here I comment some interesting scores (all up to 5 away 5 away with a non mandatory cube). The (take-/double-) points are evaluated on the basis of Ortega & Kleinman's match equity table from D. Kleinman & A. Ortega, "Backgammon With the Giants, Neil Kazaross", 2001. This and more tables can be found via the source of GNU Backgammon. Copying and distribution of verbatim and modified versions of the tables is permitted in any medium provided the copyright notice and this permission notice are preserved. TP, DP, CP, reTP and reDP is explained in the section about *the match doubling window*.

- 5 away 5 away: TP, DP(=reDP), reTP = 24.7%, 50.0%, 27.6% both ways.

  Doubling and take behavior is close to money game. This might get marginally distorted by the fact that the redoubling behavior is more aggressive and take of redoubles is more conservative. The doubled *gammon price* is 0.61. Therefore the cube have a larger value but the gammons cost more.
  If someone doubles for the match the TP is 14.8%.

- 4 away 5 away: leader TP, DP, CP, reTP, reDP = 26.4%, 49.7%, 73.3%, 29.1%, 49.0%

  The cube behavior is surpriseling symmetric at this score. Here the leader takes conservatively and the trailer doubles more aggressively (the leader's doubled *gammon price* is 0.72). On the other hand the trailer's takes conservatively (the trailer's doubled *gammon price* is 0.74) and the leader doubles aggresively on gammon chances. If the trailer has given an initial cube he has a TP on 25.2%.

- 4 away 4 away: TP, DP(=reDP), reTP = 25.0%, 50.0%, 32.2% both ways.

  Doubling behavior is aggressive and take behavior is conservative (the doubled *gammon price* is 0.90). The redoubling behavior is very aggressive and take behavior is very conservative.

- 3 away 5 away: leader TP, DP, CP, reTP, reDP = 25.3%, 46.1%, 70.5%, 33.3%, 62.8%

  At this score the trailer handles the cube a little more aggressively and the leader takes more conservatively (the leader's doubled *gammon price* is 0.71). The leader handles the cube with care and the trailer's takes are surprisingly conservative (the trailer's doubled *gammon price* is 0.42). The cube has quite a big value for him and he redoubles very aggressively. If the trailer has given an initial cube he has a TP on 14,8%.
  The verbal presentation at this score is close to the cube handling for all the scores outside 5 away 5 away although they will probably look more like money game.

- 3 away 4 away: leader TP, DP, DP*, CP, reTP, reDP = 22.8%, 39.0%, 60.8%, 64.3%, 40.2%, 70.4% (DP* includes a redouble)

  This score is very spicy and much like 2 away 4 away! The trailer's cube handling will be conservative but extremly aggressive if he has a significant gammon chance (the leader's doubled *gammon price* is 0.94).
  The leader's cube handling is conservative with gammon chances (the real double-point should maybe include an expected redouble). The trailer's TP is 35.7% but the cube has much value to him and the trailer's doubled *gammon price* is only 0.39. His redouble is almost *mandatory*. He will often have a redouble while he is actually behind in the game (has more than 29.6% winning chances and some market loosers)!
  If the trailer has given an initial cube he has a TP on 16.9%.

- 3 away 3 away: TP, DP(=reDP), reTP = 30.4%, 50.0%, 25.0% both ways.

  Doubling behavior is aggressive (the doubled *gammon price* is 0.5) and take behavior is very conservative. The redoubling behavior is like for money with no gammons and recube value.

- 2 away 5 away: leader TP, DP, CP, DP*, CP* = 19.9%, 35.1%, 63.2%, 71.9%, 78.9% (DP*, CP* includes the *mandatory redouble*)

  Both players double more conservatively and takes more than in money game. This is a little sensitive to the trailer's gammon chances (the leader's doubled *gammon price* is 0.69). Note that the leader has a surprisingly aggressive take behavior. It will often be correct for the leader to play on for an *undoubled gammon* (the trailer's *gammon price* is 0.85).

- 2 away 4 away: leader TP, DP, CP, DP*, CP* = 19.6%, 36.7%, 66.2%, 70.4%, 83.1%

  Both players double more conservatively and takes more than in money game in no gammon situations. This is very sensitive to the trailer's gammon chances. If the trailer has a gammon chance he can double very aggressively and the leader should drop more (the leader's doubled *gammon price* is 1). If the trailer mindlessly doubles on his first roll he will only give up about 2% match winning chance. Just 10% gammons to the trailer makes the leader's take behavior more conservative than in money game!
  It will often be correct for the leader to play on for an *undoubled gammon* at this score (the trailer's *gammon price* is 0.73).

- 2 away 3 away: leader TP, DP, CP, DP*, CP* = 28.6%, 44.4%, 64.3%, 66.7%, 75.0%

  The leader should double almost as in money game (the cube dies after the *mandatory redouble* but the double-point is higher than for money and gammons do not count). It will sometimes be correct for him to play on for an *undoubled gammon* (the trailer's *gammon price* is 1). On the other hand the leader takes a little more conservatively and the trailer takes almost as in money game. The trailer's doubling behavior compared to money game is a little more aggressive (the leader has no value of the cube but his doubled *gammon price* is only 0.43).

- 2 away 2 away: TP, DP = 30%, 50% both ways.

  This score should always lead to a double match point situation so you might as well double at your first legal opportunity.

- 1 away 3 or 5 away (post Crawford): TP, DP = 0%, 100%.

  The trailer's double is *mandatory*. The leader does not have a free drop.

- 1 away 2 or 4 away (post Crawford): TP, DP = 50%, 100%.

  The trailer's double is *mandatory*. The leader has a free drop.

Not surprisingly we see that it is very important to hit the target. I.e. be very conservative if you get more points than needed to win the match by doubling or taking and aggressive when you get the exact number of point needed to win the match. This conclusion can also be used outside 5 away 5 away. If you need 8 points to win the match and own the cube at 2 you should redouble aggressively on your gammon chances. If you need e.g. 5 points your gammon chances is not making your redouble more aggressive but rather more conservative (see *too good in matches*).

My advise to beginners and intermediate players is that you should almost ignore MWC and play money game on initial doubles out side leader 2 away and trailer 5 away. If the leader is 2 away he has a TP close to 20% (doubled gammon price approximately 0.6) and doubles surprisingly aggressive in no gammon situations (even with the *mandatory redouble*). In gammonish positions he often plays on for the undoubled gammon and takes conservatively.

The crawford game is very dependent on the trailer's score. If the trailer lacks an even number of points his gammon value is approximately 0.65 (with the exeption of 2 away where it is 1). If he lacks an odd number of points his value is close to 0. In the post crawford games the leader has a gammon price on 0.7.

On redoubles the trailer should take aggressive if the leader does not hit the target. Genneraly the leader should play a little more conservative depending on how big his lead is.

Large gammon prices (values) are often linked to surprisingly small take-points (large cash-points). Genneraly speeking the gammon price is larger in matches than for money. This often makes money game and match take/drop behavior similar with an insignificant tendency to drop more in matches.

The observation has more impact on doubling in matches which is more agressive since gammon prices have a great impact on the volatility of a given position. More volatility will often give more agressive doubling behavior. You more often double your opponent in (double while he has a take). On the other hand the opponent's value of cube ownership is also often large but genneraly you will see more positions with a correct double in match positions than in money game (assuming no Jacoby rule and that the cube is not dead).

If the score is such that non can get close to winning the match the cubebehavior approaches that in moneygame. There will be a distortion to that behavior if one of the parties in the match is trailing significantly. Then the trailer have lover take-point and the leader have higher take-point the higher the level of the cube.

3.7 Tables

I have listed some processed data from different match equity tables in the following tables. The original tables can be found via the source of GNU Backgammon. Copying and distribution of verbatim and modified versions of the tables is permitted in any medium provided the copyright notice and this permission notice are preserved.

Notation									
TP	DP	CP	reTP	reDP	reCP	GP	GV	Dbl GP	Dbl GV
Take-Point	*Double-Point*	*Cash-Point*	*Redouble Take-Point*	*Redouble Double-Point*	*Redouble Cash-Point*	*Gammon Price*	*Gammon Value*	*Doubled Gammon Price*	*Doubled Gammon Value*

D. Kleinman & A. Ortega, "Backgammon With the Giants, Neil Kazaross" (2001)

	TP	DP	CP	reTP	reDP	reCP	GP	GV	Dbl GP	Dbl GV
5 away 5 away	24.7%	50.0%	75.3%	27.6%	50.0%	72.4%	0.49	0.49	0.61	0.61
4 away 5 away	26.4%	49.7%	73.3%	29.1%	62.0%* (49.0%)	74.8%* (69.7%)	0.56	0.57	0.72	0.75
4 away 4 away	25.0%	50.0%	75.0%	32.2%	50.0%	67.8%	0.50	0.50	0.90	0.90
3 away 5 away	25.3%	46.1%	70.5%	33.3%	77.2%* (62.8%)	85.2%* (80.3%)	0.56	0.65	0.71	0.42
3 away 4 away	22.8%	39.0%	64.3%	40.2%	70.4%	83.1%	0.55	0.86	0.94	0.39
3 away 3 away	30.4%	50.0%	69.6%	25.0%	50.0%	75.0%	0.78	0.78	0.50	0.50
2 away 5 away	19.9%	71.9%* (35.0%)	78.9%* (63.2%)	42.6%	100.0%	100.0%	0.46	0.85	0.74	0.00
2 away 4 away	19.6%	78.0%* (36.7%)	83.1%* (66.2%)	50.0%	100.0%	100.0%	0.42	0.73	1.00	0.00
2 away 3 away	28.6%	66.7%* (44.4%)	75.0%* (64.3%)	30.0%	100.0%	100.0%	0.80	1.00	0.42	0.00
2 away 2 away	30.0%	50.0%	70.0%	0.0%	50.0%	100.0%	0.75	0.75	0.00	0.00

Snowie 2.1, Oasya (1999)

	TP	DP	CP	reTP	reDP	reCP	GP	GV	Dbl GP	Dbl GV
5 away 5 away	24.8%	50.0%	75.2%	28.9%	50.0%	71.1%	0.49	0.49	0.68	0.68
4 away 5 away	23.5%	44.4%	70.7%	30.2%	61.8%* (48.2%)	73.5%* (67.6%)	0.50	0.62	0.80	0.86
4 away 4 away	28.0%	50.0%	72.0%	33.6%	50.0%	66.4%	0.64	0.64	1.02	1.02
3 away 5 away	21.0%	40.0%	68.5%	33.2%	76.1%* (61.2%)	84.3%* (79.0%)	0.44	0.66	0.72	0.46
3 away 4 away	22.8%	37.8%	62.5%	40.6%	69.2%	81.9%	0.57	0.95	0.98	0.44
3 away 3 away	31.0%	50.0%	69.0%	25.2%	50.0%	74.8%	0.82	0.82	0.51	0.51
2 away 5 away	17.2%	69.0%* (30.8%)	77.1%* (61.3%)	40.7%	100.0%	100.0%	0.39	0.88	0.69	0.00
2 away 4 away	18.8%	76.6%* (34.2%)	81.9%* (63.8%)	50.0%	100.0%	100.0%	0.42	0.80	1.00	0.00
2 away 3 away	27.0%	66.5%* (42.3%)	74.8%* (63.2%)	31.5%	100.0%	100.0%	0.75	1.02	0.46	0.00
2 away 2 away	31.5%	50.0%	68.5%	0.0%	50.0%	100.0%	0.85	0.85	0.00	0.00

J. Jacobs & W. Trice, Can a Fish Taste Twice as Good (1996)

	TP	DP	CP	reTP	reDP	reCP	GP	GV	Dbl GP	Dbl GV
5 away 5 away	24.4%	50.0%	75.6%	27.6%	50.0%	72.4%	0.47	0.47	0.62	0.62
4 away 5 away	23.8%	44.4%	70.2%	29.6%	63.0%* (49.6%)	75.3%* (70.0%)	0.51	0.64	0.73	0.74
4 away 4 away	28.8%	50.0%	71.2%	31.4%	50.0%	68.6%	0.68	0.68	0.84	0.84
3 away 5 away	23.0%	46.5%	73.5%	33.2%	76.2%* (61.4%)	84.4%* (79.1%)	0.45	0.52	0.72	0.45
3 away 4 away	24.2%	43.1%	68.0%	39.7%	69.0%	82.2%	0.55	0.73	0.93	0.42
3 away 3 away	29.2%	50.0%	70.8%	25.2%	50.0%	75.8%	0.70	0.70	0.51	0.51
2 away 5 away	20.9%	70.7%* (34.7%)	77.4%* (60.7%)	42.5%	100.0%	100.0%	0.53	0.99	0.74	0.00
2 away 4 away	20.6%	77.2%* (36.7%)	82.2%* (64.4%)	50.0%	100.0%	100.0%	0.47	0.81	1.00	0.00
2 away 3 away	27.5%	66.5%* (43.0%)	74.8%* (63.5%)	31.0%	100.0%	100.0%	0.77	1.02	0.45	0.00
2 away 2 away	31.0%	50.0%	69.0%	0.0%	50.0%	100.0%	0.82	0.82	0.00	0.00

N. Zadeh, Management Science 23, 986 (1977)

	TP	DP	CP	reTP	reDP	reCP	GP	GV	Dbl GP	Dbl GV
5 away 5 away	24.9%	50.0%	75.2%	28.4%	50.0%	71.6%	0.49	0.49	0.66	0.66
4 away 5 away	23.7%	44.8%	70.7%	30.6%	62.4%* (49.9%)	74.5%* (69.3%)	0.51	0.62	0.79	0.80
4 away 4 away	28.5%	50.0%	71.5%	32.2%	50.0%	67.8%	0.67	0.67	0.91	0.91
3 away 5 away	22.0%	42.5%	70.3%	33.9%	76.7%* (62.8%)	84.8%* (79.9%)	0.45	0.61	0.74	0.44
3 away 4 away	22.9%	39.2%	64.4%	40.1%	70.4%	83.1%	0.55	0.86	0.94	0.39
3 away 3 away	30.8%	50.0%	69.2%	24.4%	50.0%	75.6%	0.80	0.80	0.48	0.48
2 away 5 away	19.7%	71.9%* (34.2%)	78.6%* (62.1%)	43.7%	100.0%	100.0%	0.47	0.89	0.77	0.00
2 away 4 away	19.7%	78.0%* (36.8%)	83.0%* (66.2%)	50.0%	100.0%	100.0%	0.42	0.73	1.00	0.00
2 away 3 away	29.8%	67.2%* (46.6%)	75.6%* (65.8%)	28.8%	100.0%	100.0%	0.83	0.95	0.40	0.00
2 away 2 away	28.8%	50.0%	71.3%	0.0%	50.0%	100.0%	0.68	0.68	0.00	0.00

Points marked with * includes a mandatory redouble. The figures in parantheses are only relevant in last roll positions.

Observe that different tables give significantly different take-points. Unfortunately the only thing we know is that non of them are likely to be correct. It is usefull to use them anyway since they are as good an estimate as any. So is the following estimates made by me on the basis of the table data above.

3.8 My Take-points

M@xFriis' subjective easier learned take-points (2003)

	TP	CP	reTP	reCP	Dbl GP	Dbl GV
5 away 5 away	1/4	3/4	2/7	5/7	3/5	3/5
4 away 5 away	1/4	5/7	3/10	3/4*	3/4	3/4
4 away 4 away	2/7	5/7	1/3	2/3	9/10	9/10
3 away 5 away	2/9	7/10	1/3	5/6*	7/10	3/7
3 away 4 away	2/9	2/3	2/5	5/6	9/10	2/5
3 away 3 away	3/10	7/10	1/4	3/4	1/2	1/2
2 away 5 away	1/5	4/5*	3/7	1	3/4	0
2 away 4 away	1/5	5/6*	1/2	1	1	0
2 away 3 away	2/7	3/4*	3/10	1	3/7	0
2 away 2 away	3/10	7/10	0	1	0	0
Alternative round	1/6 = 6/36, 1/5 ≈ 7/36, 2/9 = 8/36, 1/4 = 9/36, 2/7 ≈ 10/36, 3/10 ≈ 11/36, 1/3 = 12/36, etc.					

M@xFriis' subjective easier used take-points (2003)

	TP	1-CP	reTP	1-reCP	Dbl GP	Dbl GV
5 away 5 away	9/36	9/36	10/36	10/36	3/5	3/5
4 away 5 away	9/36	10/36	11/36	9/36*	3/4	3/4
4 away 4 away	10/36	10/36	12/36	12/36	9/10	9/10
3 away 5 away	8/36	11/36	12/36	6/36*	7/10	3/7
3 away 4 away	8/36	12/36	14/36	6/36	9/10	2/5
3 away 3 away	11/36	11/36	9/36	9/36	1/2	1/2
2 away 5 away	7/36	7/36*	15/36	0	3/4	0
2 away 4 away	7/36	6/36*	18/36	0	1	0
2 away 3 away	10/36	9/36*	11/36	0	3/7	0
2 away 2 away	11/36	11/36	0	0	0	0

Points marked with * includes a mandatory redouble.

Some of the leaders points and prices are reasonably stable for scores outside trailer 5 away.

- 1 away even away: GP = 2/3.
- 1 away odd away: GP = 0.
- 1 away 5+ away: Dbl GP = 7/10 (Post crawford).
- 2 away 6+ away: TP, Dbl GP = 1/5, 3/5.
- 3 away 6+ away: TP, Dbl GP = 2/9, 3/5.

This does not mean that the trailer's cubebehavior does not depend on how far he is trailing. Even though the leader has the same take-point the trailer's marked losers will cost more the more he is trailing.

You can access *my subjective table data* and the comments in *the example section* wireless at http://xfriis.dk/wap/bg.wml.

Finally I have made a more *printable version of this article*. Comments can be send to m@xfriis.dk